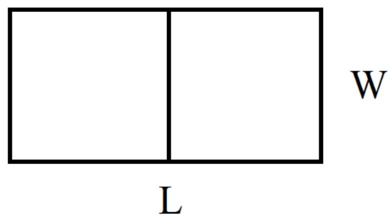
Exercise 86

Find the dimensions of the rectangular corral split into 2 pens of the same size producing the greatest possible enclosed area given 300 feet of fencing.

Solution

Draw a schematic of the rectangular corral, labelling the length and width as L and W, respectively.



The perimeter is the sum of the lengths.

$$\begin{aligned} P &= L + L + W + W + W \\ &= 2L + 3W \end{aligned}$$

It's given to be 300 feet.

$$300 = 2L + 3W$$

Solve for L.

$$300 - 3W = 2L$$

$$\frac{1}{2}(300 - 3W) = L$$

$$L = 150 - \frac{3}{2}W$$

Write the formula for the area, substitute the result for the length, and complete the square to write the quadratic function in vertex form.

$$\begin{aligned} A = LW = \left(150 - \frac{3}{2}W\right)W &= 150W - \frac{3}{2}W^2 \\ &= -\frac{3}{2}(W^2 - 100W) \\ &= -\frac{3}{2}[(W^2 - 100W + 50^2) - 50^2] \\ &= -\frac{3}{2}[(W - 50)^2 - 50^2] \\ &= -\frac{3}{2}(W - 50)^2 + 3750 \end{aligned}$$

Therefore, the maximum area is $A = 3750$ ft^2, which occurs when $W = 50$ ft and $L = 150 - \frac{3}{2}(50) = 75$ ft.